# A COMPARISON OF SOME EQUATIONS IN CLASSICAL, RELATIVISTIC AND AN ALTERNATIVE ELECTRODYNAMICS FOR AN ELECTRON MOVING IN AN ELECTRIC FIELD 

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#### Abstract

A comparison of some equations is made in classical, relativistic and an alternative electrodynamics. Motion of an electron, in an electric field, is treated under acceleration or deceleration or circular motion. At low speeds, relativistic and the alternative electrodynamics converge to classical electrodynamics. Classical electrodynamics is incompatible with relativistic and the alternative electrodynamics at high speeds near that of light. Relativistic and the alternative electrodynamics show agreement for accelerated electrons, but there is divergence for decelerated electrons. Considering aberration of electric field, equations of motion of the alternative electrodynamics are derived for charged particles moving up to the speed of light, with constant mass and emission of radiation. Revolution of an electron, round a central force of attraction, is shown to be stable outside quantum mechanics.


Keywords: Aberration, acceleration, electric charge, field, energy, light, mass, radiation, radius.

## 1. Introduction

There are now three systems of electrodynamics that happen to be applicable under different situations. Classical electrodynamics [1, 2, 3] is applicable to electrically charged particles, such as an electron, moving at very low speeds compared to the speed of light in a vacuum. Relativistic electrodynamics [4, 5] is for charged particles moving at speeds near that of light. Quantum electrodynamics [6, 7] is for atomic particles moving at high speeds.

The alternative electrodynamics [8] incorporates Newton's laws of motion and extends Coulomb's law of electrostatic force, through aberration of electric field, for a moving electron. It is a consistent system applicable to all charged particles, moving at speeds up to that of light, with emission of radiation and mass remaining constant.

In classical electrodynamics the force exerted by an electric field on a moving electon is independent of speed of the particle and it can be accelelerate to a speed beyond that of light, if the force acts long enough. In relativistic electrodynamics the mass of a particle is supposed to increase with its speed, becoming infinitely large at the speed of light. This then becomes a speed limit, as an infinite mass cannot be accelerated any faster by a force. In the alternative electrodynamics accelerating force decreases with speed becoming zero at the speed of light as a limit. The existence of such a limiting speed was demonstrated by Bertozzi's experiment in 1964 [9].

The main differeces between the classical, the relativistic and the alternative electrodynamics are with respect to decelerated electrons and emission of radiation. In the relativistic elctrodynamics, an electron moving at the speed of light cannot be stopped by any retarding force and there is no radiation reaction force. In the alternative electrodynamics an electron moving at the speed of light, with constant mass, is easily stopped by a retarding field, with emission of radiation.

In classical electrodynamics radiation reaction force is proportional to the rate of change of acceleration (Abraham-Lorentz formula) [10] and radiation power is proportional to the square of acceleration (Larmor formula) [10]. There is no formula for radiation reaction force in relativistic electrodynamics but there is an expression for radiation power (Lienard formula) [10]. In the alternative electrodynamics radiation reaction force is proportional to the velocity in an electric field and radiation power is the scalar product of radiation force and velocity. It is aberration of electric field and radiation power that make the difference between the 3 systems of electrodynamics.

## 2. Aberration of Electric Field

Aberration of electric field is a phenomenon similar to aberration of light discovered by English astronomer, James Bradley, in 1728 [11]. Figure 1 depicts an electron of mass $m$ and charge $-e$ moving at $P$ with velocity $\mathbf{v}$ at an angle $\theta$ to the force of attraction due to an electric field of intensity $\mathbf{E}$ from a stationary source charge $+Q$ at $O$. The electron is subjected to aberration of electric field whereby the direction of propagation of the force of attraction, given by velocity vector $\mathbf{c}$, is displaced from the the instantaneous line $P O$ through angle of aberration $\alpha$, such that:


Figure 1. Depicting an electron of mass $m$ and charge $-e$ at a point $P$ moving with velocity $\mathbf{v}$ at angle $\theta$ to force of attraction $\mathbf{F}$ due to an electric field of intensity $\mathbf{E}$ from a stationary source charge $Q$ at $O$.

$$
\begin{equation*}
\sin \alpha=\frac{v}{c} \sin \theta \tag{1}
\end{equation*}
$$

### 2.1 Relative Velocity of Propagation of Electric Force

With reference to Figure 1, the vector $\mathbf{z}=(\mathbf{v}-\mathbf{c})$ is the relafive velocity of transmission between the electric force propagated with velocity of light $\mathbf{c}$ and the electron moving with velocity $\mathbf{v}$, thus:

$$
\begin{equation*}
\mathbf{z}=(\mathbf{c}-\mathbf{v})=-\sqrt{c^{2}+v^{2}-2 c v \cos (\theta-\alpha)} \hat{\mathbf{u}} \tag{2}
\end{equation*}
$$

where $(\theta-\alpha)$ is the angle between the vectors $\mathbf{c}$ and $\mathbf{v}$ and $\hat{\mathbf{u}}$ is a unit vector in the positive direction of the field $\mathbf{E}$. The electron can move with $\theta=0, \pi$, or $\pi / 2$ radians.

With $\theta=0$ there is motion in a straight line with acceleration and equations (1) and (2) give the relative speed of transmission of the electric force as:

$$
\begin{equation*}
z=c-v \tag{3}
\end{equation*}
$$

Where $\theta=\pi$ radians, there is motion in a straight line with deceleration and the relative speed is:

$$
\begin{equation*}
z=c+v \tag{4}
\end{equation*}
$$

If $\theta=\pi / 2$ radians, and noting that $\sin \alpha=v / c$, there is circular revolution with constant speed $v$, giving the speed of transmission of the force as:

$$
\begin{equation*}
z=\sqrt{c^{2}-v^{2}} \tag{5}
\end{equation*}
$$

Equatons (3), (4) and (5) demonstrate the relativity of speed of light with respect to an observer or an object moving at speed $v$.

### 2.2 Accelerating Force on a Moving Electron

Accelerating force $\mathbf{F}$ at velocity $\mathbf{v}$ and time $t$, in a field of magnitude $E$, is put as:

$$
\begin{equation*}
\mathbf{F}=\frac{e E}{c}(\mathbf{c}-\mathbf{v})=-\frac{e E}{c} \sqrt{c^{2}+v^{2}-2 c v \cos (\theta-\alpha)} \hat{\mathbf{u}}=m \frac{d \mathbf{v}}{d t} \tag{6}
\end{equation*}
$$

where $\hat{\mathbf{u}}$ is a unit vector in the positive direction of the electric field $\mathbf{E}$. (Fig. 1) For accelerated motion in a straight line, with $\theta=0$, equations (1) and (6) give the vector:

$$
\begin{equation*}
-F \hat{\mathbf{u}}=-e E\left(1-\frac{v}{c}\right) \hat{\mathbf{u}}=-m \frac{d v}{d t} \hat{\mathbf{u}} \tag{7}
\end{equation*}
$$

The scalar first order differential equation of motion, for an accelerated electron, is:

$$
\begin{equation*}
F=e E\left(1-\frac{v}{c}\right)=m \frac{d v}{d t} \tag{8}
\end{equation*}
$$

For decelerated motion in a straight line, with $\theta=\pi$ radians, equations (1) and (6) give:

$$
\begin{equation*}
-F \hat{\mathbf{u}}=-e E\left(1+\frac{v}{c}\right) \hat{\mathbf{u}}=m \frac{d v}{d t} \hat{\mathbf{u}} \tag{9}
\end{equation*}
$$

The scalar first order differential equation of motion, for a decelerated electron, is:

$$
\begin{equation*}
F=e E\left(1+\frac{v}{c}\right)=-m \frac{d v}{d t} \tag{10}
\end{equation*}
$$

Where $\theta=\pi / 2$ radians there is motion in a circle of radius $r$ with constant speed $v$ and centripetal acceleration $-v^{2} / r$. Equation (1) and (6) give the vector equation:

$$
\begin{equation*}
-F \hat{\mathbf{u}}=-e E \sqrt{1-\frac{v^{2}}{c^{2}}} \hat{\mathbf{u}}=-m \frac{v^{2}}{r} \hat{\mathbf{u}}=m_{o} \frac{v^{2}}{r} \hat{\mathbf{u}} \tag{11}
\end{equation*}
$$

where $m=m_{o}$, the rest mass. The scalar equation is:

$$
\begin{equation*}
F=e E \sqrt{1-\frac{v^{2}}{c^{2}}}=m \frac{v^{2}}{r}=m_{o} \frac{v^{2}}{r} \tag{12}
\end{equation*}
$$

Equation (8), (10) and (12) show the dependence of accelerating force $F$ on speed $v$.
Relativistic electrodynamics regards the accelerating force in equation (12) as constant at $-e E$, independent of speed, while mass $m$ varies with speed $v$ as:

$$
\begin{equation*}
m=\frac{m_{o}}{\sqrt{1-\frac{v^{2}}{c^{2}}}}=\gamma m_{o} \tag{13}
\end{equation*}
$$

where $\gamma$ is the Lorentz factor. Equations (1) to (13) are used in this paper to make comparisons between the three systems of electrodybanic. It is dependence of accelerating force on speed and energy radiation that make all the difference.

## 3. Comparison of Equations in $\mathbf{3}$ Systems of Electrodynamics

A comparison of some equations in (A) classical, (B) relativistic and (C) an alternative electrodynamics is given in the Table 1 below. Vector quantities are indicated in boldface type and scalar quantities in ordinary type. Other Issues included are Speed of Light in a Moving Medium [12], Fringe Shifts in the MichelsonMorley and Sagnac Experiments [13,14] and Doppler Shift in Light and Sound.

| A. CLASSICAL ELECTRODYNAMICS | B. RELATIVISTIC ELECTRODYNAMICS | C. ALTERNATIVE ELECTRODYNAMICS |
| :---: | :---: | :---: |
| 1. CHARGE OF A MOVING ELECTRON Magnitude of charge of an electron -e is independent of speed $v$ of the electron at time $t$ : $e=e_{o}$ (rest charge) | 1. CHARGE OF A MOVING ELECTRON <br> Magnitude of charge of an electron -e is independent of speed $v$ of the electron at time $t$ : $e=e_{0}$ (rest charge) | 1. CHARGE OF A MOVING ELECTRON <br> Magnitude of charge of an electron -e is independent of speed $v$ of the electron at time $t$ : $e=e_{0}$ (rest charge) |
| 2. MASS OF A MOVING <br> ELECTRON <br> Mass $m$ of an electron is independent of speed $v$ of the particle at time $t$ : $m=m_{0}$ where $m_{0}$ is the rest mass (at $v=$ 0 ) with respect to an observer). | 2. MASS OF A MOVING <br> ELECTRON <br> Mass $m$ varies with speed $v$ as: $m=m_{0}\left(1-\frac{v^{2}}{c^{2}}\right)^{-\frac{1}{2}}$ <br> where $m_{0}$ is rest mass at $v=0$. | 2. MASS OF A MOVING <br> ELECTRON <br> Mass $m$ of an electron is independent of speed $v$ of the particle at time $t$ : $m=m_{0}$ where $m_{0}$ is the rest mass (at $v=$ 0) with respect to an observer) |
| 3. RELATIVE VELOCITY <br> Relative velocity $\boldsymbol{w}$ of one body moving at velocity u with respect to an observer (another body) | 3. RELATIVE VELOCITY <br> Relative velocity $w$ of one body moving at velocity u with respect to an observer (another body) | 3. RELATIVE VELOCITY <br> Relative velocity $w$ of one body moving at velocity u with respect to an observer (another body) |


| moving with velocity $\mathbf{v}$, is: $\mathbf{w}=\mathbf{u}-\mathbf{v}$ <br> where $\mathbf{u}$ and $\mathbf{v}$ are vector velocities in any direction. | moving with velocity $\mathbf{v}$, is: $\mathbf{w}=(\mathbf{u}-\mathbf{v})\left(1-\frac{u v}{c^{2}}\right)^{-1}$ <br> where $\mathbf{u}$ and $\mathbf{v}$ are collinear. | moving with velocity $\mathbf{v}$, is: $\mathbf{w}=\mathbf{u}-\mathbf{v}$ <br> where $\mathbf{u}$ and $\mathbf{v}$ are vector velocities in any direction. |
| :---: | :---: | :---: |
| 4. ADDITION OF VELOCITIES The velocity w of a passenger moving with velocity $\mathbf{u}$ in a ship that is cruising with velocity $\mathbf{s}$ relative to an observer moving with velocity $\mathbf{v}$ with espect to a frame of reference (a point on the seashore) is: $\mathbf{w}=\mathbf{s}+\mathbf{u}-\mathbf{v}$ <br> where $\mathbf{s}, \mathbf{u}$ and $\mathbf{v}$ are vector velocities in any direction. | 4. ADDITION OF VELOCITIES The velocity $\mathbf{w}$ of a passenger moving with velocity $\mathbf{u}$ in a ship that is cruising with velocity $\mathbf{s}$ relative to an observer moving with velocity $\mathbf{v}$ with espect to a frame of reference (a point on the seashore) is: $\mathbf{w}=(\mathbf{s}+\mathbf{u})\left(1+\frac{s u}{c^{2}}\right)^{-1}$ <br> where $\mathbf{s}$ and $\mathbf{u}$ are collinear. | 4. ADDITION OF VELOCITIES The velocity w of a passenger moving with velocity $u$ in a ship that is cruising with velocity $\mathbf{s}$ relative to an observer moving with velocity $\mathbf{v}$ with espect to a frame of reference (a point on the seashore) is: $\mathbf{w}=\mathbf{s}+\mathbf{u}-\mathbf{v}$ <br> where $\mathbf{s}, \mathbf{u}$ and $\mathbf{v}$ are vector velocities in any direction. |
| 5. VELOCITY OF LIGHT RELATIVE TO AN OSERVER <br> 5.1. Velocity of light $\mathbf{z}$ from a source moving with velocity $\mathbf{u}$, as seen by an observer moving with velocity $\mathbf{v}$, with respect to a frame of reference), is vector: $\mathbf{z}=\mathbf{c}+(\mathbf{u}-\mathbf{v})$ <br> where $c$ is the speed of light, a constant, relative to the source. <br> 5.2. For a stationary source ( $u=$ 0 ), the speed of light, with respect to a body moving in the direction of propagation of the ray, is a scalar: $z=c-v$ <br> 5.3. For a stationary source ( $u=$ <br> 0 ), the speed of light, with respect to a body moving in the opposite direction of propagation of the ray, is: $z=c+v$ <br> 5.4. For a stationary source ( $u=$ 0), the speed of light, with respect to a body moving perpendicular to the direction of propagation of the ray, is: $z= \pm \sqrt{c^{2}+v^{2}}$ <br> No consideration is given to aberration of light. | 5. VELOCITY OF LIGHT RELATIVE TO AN OSERVER <br> 5.1. Speed of light $z$ from a source moving with velocity $\mathbf{u}$, as seen by an observer moving with velocity $\mathbf{v}$, with respect to a frame of reference), is: $z=c$ <br> where $c$ is the speed of light, a constant, relative to the source. <br> 5.2. For a stationary source ( $u=$ <br> 0 ), the speed of light, with respect to a body moving in the direction of propagation of the ray, is a scalar: $z=c$ <br> 5.3. For a stationary source ( $u=$ 0 ), the speed of light, with respect to a body moving in the opposite direction of propagation of the ray, is: $z=c$ 5.4. For a stationary source ( $u=$ 0 ), the speed of light, with respect to a body moving perpendicular to the direction of propagation of the ray, is: $z= \pm c$ <br> No consideration is given to aberration of light.. | 5. VELOCITY OF LIGHT RELATIVE TO AN OSERVER <br> 5.1. Velocity of light $\mathbf{z}$ from a source moving with velocity $\mathbf{u}$, as seen by an observer moving with velocity $\mathbf{v}$, with respect to a frame of reference), is vector: $\mathbf{z}=\mathbf{c}+(\mathbf{u}-\mathbf{v})$ <br> where $c$ is the speed of light, a constant, relative to the source. <br> 5.2. For a stationary source ( $u=$ 0 ), the speed of light, with respect to a body moving in the direction of propagation of the ray, is a scalar: $z=c-v$ <br> 5.3. For a stationary source ( $u=$ <br> 0 ), the speed of light, with respect to a body moving in the opposite direction of propagation of the ray, is: $z=c+v$ <br> 5.4. For a stationary source ( $u=$ 0 ), the speed of light, with respect to a body moving perpendicular to the direction of propagation of the ray, is: $z= \pm \sqrt{c^{2}-v^{2}}$ <br> Consideration is given to aberration of light.. |
| 6. VELOCITY OF PROPAGATION OF AN ELECTRIC FORCE | 6. VELOCITY OF PROPAGATION OF AN ELECTRIC FORCE | 6. VELOCITY OF PROPAGATION OF AN ELECTRIC FORCE |

6.1. Velocity of propagation of electric force from a stationary source charge is: $\infty$ (infinite) 6.2. Velocity of transmission of an electric force, relative to a charged particle moving with velocity $\mathbf{v}$, is: $\infty$ (infinite)
6.3. Speed of transmission of an electric force in rectilinear motion (in the direction of force) with acceleration, is: $\infty$ (infinite)
6.4. Speed of transmission of an electric force in rectilinear motion (against the edirection of the foce) with deceleration, is:
$\infty$ (infinite)
6.5. Speed of transmission of an electric force in circular motion (perpendicular to the force) with centripetal acceleration, is:
$z= \pm \sqrt{c^{2}+v^{2}}$
7. NEWTON'S SECOND LAW OF MOTION:
Accelerating force $\mathbf{F}=m \frac{d \mathbf{v}}{d t}$ where $\mathbf{v}$ is the velocity and $\frac{d \mathbf{v}}{d t}$ the acceleration of a constant mass $m$, at time $t$.
8. COULOMB'S LAW ON A MOVING ELECTRON
8.1. Consider an electron of charge $-e$ and mass $m$, moving with velocity $\mathbf{v}$, at time $t$, under the acceleration ( $d \mathbf{v} / d t$ ) of an electric field of intensity $\mathbf{E}$.
8.2. For a stationary source charge $Q$, electrostatic force is:

$$
\begin{aligned}
& \mathbf{F}_{o}=\frac{-Q e}{4 \pi \varepsilon_{o} r^{2}} \hat{\mathbf{u}}=-e \mathbf{E} \\
& \mathbf{E}=\frac{Q}{4 \pi \varepsilon_{o} r^{2}} \hat{\mathbf{u}}
\end{aligned}
$$

where $\mathbf{E}$ is the electric field intensity of magnitude $E, \varepsilon_{0}$ is the permittivity of a vacuum, $r$ is the distance between the
6.1. Velocity of propagation of an electric force from a stationary source charge is: $\infty$ (infinite) 6.2. Velocity of transmission of an electric force, relative to a charged particle moving with velocity $\mathbf{v}$, is: $\mathbf{c}$ (velocity of light) 6.3. Speed of transmission of an electric force in rectilinear motion (in the direction of the force) with acceleration, is: c (speed of light) 6.4. Speed of transmission of an electric force in rectilinear motion (against the edirection of the force) with deceleration, is:
c (speed of light)
6.5. Speed of transmission of an electric force in circular motion (perpendicular to the force) with centripetal acceleration, is: $\pm c$ (speed of light)
7. NEWTON'S SECOND LAW OF MOTION:
Accelerating force $\mathbf{F}=\frac{d}{d t}(m \mathbf{v})$ where $\mathbf{v}$ is the velocity and $(m \mathbf{v})$ the momentum of a mass $m$, which depends on speed $v$ at time $t$
8. COULOMB'S LAW ON A MOVING ELECTRON
8.1. Consider an electron of charge $-e$ and mass $m$, moving with velocity $\mathbf{v}$, at time t , under the acceleration ( $d \mathrm{v} / \mathrm{dt}$ ) of an electric field of intensity $\mathbf{E}$.
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$\mathbf{E}=\frac{Q}{4 \pi \varepsilon_{o} r^{2}} \hat{\mathbf{u}}$
where $\mathbf{E}$ is he electric field intensity of magnitude $E, \varepsilon_{0}$ is the permittivity of a vacuum, $r$ is the distance between the
6.1. Velocity of propagation of an electric force from a fixed source charge is velocity of light: $\mathbf{z = c}$ 6.2. Velocity of transmission of an electric force, relative to a charged particle moving with velocity $\mathbf{v}$, is vector: $\mathbf{z =} \mathbf{c} \mathbf{- v}$
6.3. Speed of transmission of an electric force in rectilinear motion (in the direction of the force) with acceleration, is: $z=c-v$
6.4. Speed of transmission of an electric force in rectilinear motion (against the edirection of the force) with deceleration, is:
$z=c+v$
6.5. Speed of transmission of an electric force in circular motion (perpendicular to the force) with centripetal acceleration, is:

$$
z= \pm \sqrt{c^{2}-v^{2}}
$$

7. NEWTON'S SECOND LAW OF MOTION:
Accelerating force $\mathbf{F}=m \frac{d \mathbf{v}}{d t}$ where $\mathbf{v}$ is the velocity and $\frac{d \mathbf{v}}{d t}$ the acceleration of a constant mass $m$, at time $t$.
8. COULOMB'S LAW ON A MOVING ELECTRON
8.1. Consider an electron of charge -e and mass $m$, moving with velocity $\mathbf{v}$, at time t , under the acceleration ( $d \mathrm{dv} / \mathrm{dt}$ ) of an electric field of intensity $\mathbf{E}$.
8.2. For a stationary source charge $Q$, electrostatic force is:

$$
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& \mathbf{E}=\frac{Q}{4 \pi \varepsilon_{o} r^{2}} \hat{\mathbf{u}}
\end{aligned}
$$

where $\mathbf{E}$ is the electric field intensity of magnitude $E, \varepsilon_{0}$ is the permittivity of a vacuum, $r$ is the distance between the
charges ( $Q$ and $-e$ ) and $\hat{u}$ a unit vector in the positive direction of vector $\mathbf{E}$, electrostatic field.
8.3. Accelerating force, equal to the electrostatic force $F_{0}$, is:
$\mathbf{F}_{o}=-e \mathbf{E}=m \frac{d \mathbf{v}}{d t}$
No consideration of relative velocity between the electric force propagated with infinte velocity and the electron moving with velocity $\mathbf{v}$ in the field.
8.4. In rectilinear motion with acceleration, the vectors $\mathbf{E}$ and $\mathbf{v}$ are collinear and In opposite directions so that the above equation gives the vector:
$\mathbf{F}_{o}=-e E \hat{\mathbf{u}}=-m \frac{d v}{d t} \hat{\mathbf{u}}$
and the scalar equation is:

$$
\begin{equation*}
F_{o}=e E=m \frac{d v}{d t} \tag{14A}
\end{equation*}
$$

8.5. In rectilinear motion with deceleraton, the vectors $\mathbf{E}$ and $\mathbf{v}$ are collinear and in the same direction so that we get the vector equation:

$$
\mathbf{F}_{o}=-e E \hat{\mathbf{u}}=m \frac{d v}{d t} \hat{\mathbf{u}}
$$

and the scalar equation is:
$F_{o}=e E=-m \frac{d v}{d t}$
8.6. For an electron of mass $m$ and charge -e revolving in a radial electrid field of magnitude $E$ and intensity $\hat{u} E$ in a circle of radius $r$ with constant speed $v$ and centripetal acceleration $-\mathrm{u} v^{2} / r$, the accelerating force is:

$$
\mathbf{F}_{o}=-e E \hat{\mathbf{u}}=-m \frac{v^{2}}{r} \hat{\mathbf{u}}
$$

Equal to the electrostatic force.
$F_{o}=e E=m \frac{v^{2}}{r}$
8.7. Radius of revolution is:
charges ( $Q$ and -e) and û a unit vector in the positive direction of vector E , electrostatic field.
8.3. Accelerating force, equal to the electrostatic force $F_{0}$, is:
$\mathbf{F}_{o}=-e \mathbf{E}=\frac{d}{d t}(m \mathbf{v})$
No consideration of relative velocity between the electric force propagated with velocity of light $\mathbf{c}$ and the electron moving with velocity v in the field.
8.4. In rectilinear motion with acceleration, the vectors $\mathbf{E}$ and $\mathbf{v}$ are collinear and In opposite directions so that the above equation gives the vector:
$\mathbf{F}_{o}=-e E \hat{\mathbf{u}}=-\frac{d}{d t}(m v) \hat{\mathbf{u}}$ and the scalar equation is:

$$
\begin{equation*}
F_{o}=e E=\frac{d}{d t}(m v) \tag{14B}
\end{equation*}
$$

8.5. In rectilinear motion with deceleraton, the vectors $\mathbf{E}$ and $\mathbf{v}$ are collinear and in the same direction so that we get the vector equation:
$\mathbf{F}_{o}=-e E \hat{\mathbf{u}}=\frac{d}{d t}(m v) \hat{\mathbf{u}}$
and the scalar equation is:
$F_{o}=e E=-\frac{d}{d t}(m v)$
8.6. For an electron of mass $m$ and charge -e revolving in a radial electrid field of magnitude $E$ and intensity $\mathbf{u} E$ in a circle of radius $r$ with constant speed $v$ and centripetal acceleration - $\mathbf{u} \mathrm{v}^{2} / r$, the accelerating force is:
$\mathbf{F}_{o}=-e E \hat{\mathbf{u}}=-m_{o}\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}} \frac{v^{2}}{r} \hat{\mathbf{u}}$
$F_{o}=e E=m_{o}\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}} \frac{v^{2}}{r}$
8.7. Radius of revolution is:
charges ( $Q$ and $-e$ ) and û a unit vector in the positive direction of vector $\mathbf{E}$, electrostatic field.
8.3. Accelerating force $F$, not equal to electrostatic force $F_{0}$, is:
$\mathbf{F}=\frac{e E}{c}(\mathbf{c}-\mathbf{v})=m \frac{d \mathbf{v}}{d t}$
where $(\mathbf{c}-\mathbf{v})$ is the relative velocity between the electric force propagated at velocity of light $\mathbf{c}$ and the electron moving with velocity $\mathbf{v}$ in the field.
8.4. In rectilinear motion with acceleration, the vectors $\mathbf{E}$ and $\mathbf{v}$ are collinear and In opposite directions so that the above equation gives the vector:
$\mathbf{F}=-e E\left(1-\frac{v}{c}\right) \hat{\mathbf{u}}=-m \frac{d v}{d t} \hat{\mathbf{u}}$
and the scalar equation is:
$F=e E\left(1-\frac{v}{c}\right)=m \frac{d v}{d t}$
8.5. In rectilinear motion with deceleraton, the vectors $\mathbf{E}$ and $\mathbf{v}$ are collinear and in the same direction so that we get the vector equation:
$\mathbf{F}=-e E\left(1+\frac{v}{c}\right) \hat{\mathbf{u}}=m \frac{d v}{d t} \hat{\mathbf{u}}$
and the scalar equation is:
$F=e E\left(1+\frac{v}{c}\right)=-m \frac{d v}{d t}(15 \mathrm{C})$
8.6. For an electron of mass $m$ and charge -e revolving in a radial electrid field of magnitude $E$ and intensity $\mathrm{u} E$ in a circle of radius $r$ with constant speed $v$ and centripetal acceleration -ûv $2 / r$, the accelerating force is:
$\mathbf{F}=-e E\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{1}{2}} \hat{\mathbf{u}}=-m \frac{v^{2}}{r} \hat{\mathbf{u}}$
$F=e E\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{1}{2}}=m \frac{v^{2}}{r}$
8.7. Radius of revolution is:
$r=\frac{m v^{2}}{e E}=r_{o}$
(Classical radius of revolution)
8.8. For accelerated rectilinear motion with zero initial speed in a uniform electric field of magnitude $E$, with constant mass $m$ and charge -e equation (14A) gives speed $v$ at time $t$ as:
$v=a t$
where $a=\frac{e E}{m}$ is a constant.
Maximum attainable speed, as
$t \rightarrow \infty$, is infinitely large,
contrary to observations
8.9. For decelerated rectilinear motion with initial speed of light $c$ in a uniform electric field of magnitude $E$, with constant mass $m$ and charge $-e$, equation (15A) gives speed $v$ at time $t$ as: $v=c-a t$
where $a=\frac{E q}{m}$ constant.
Maximum speed, as time
$t \rightarrow \infty$, is minus infinity ( $-\infty$ ).
9. KINETIC ENERGY GAINED, POTENTIAL ENERGY LOST AND ENERGY RADIATED
9.1. Equation ( 14 A ) gives the kinetic energy $K$ gained by an electron, of mass $m$ and charge $-e$, acelerated in a straight line through a distance $x$ by an electric field $E$, from 0 (zero) initial speed to speed $v$, as: $K=$ $\int_{0}^{x} e E(d x)=m \int_{0}^{v} v \frac{d v}{d x}(d x)=\frac{1}{2} m v^{2}$

Equal to potential energy $P$ lost.
9.2. Potential energy $P$ lost is:
$P=\frac{1}{2} m v^{2}$
$r=\frac{m_{o} v^{2}}{e E}\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}}=\gamma r_{o}$
8.8. For accelerated rectilinear motion with zero initial speed in a uniform electric field of magnitude $E$, with constant mass $m$ and charge $-e$, equation (14B) gives speed $v$ at time $t$ as:
$v=a t\left\{1+\left(\frac{a t}{c}\right)^{2}\right\}^{\frac{-1}{2}}$
where $a=\frac{e E}{m}$ is a constant..
Speed of light $c$ is the maximum attainable, as time $t \rightarrow \infty$
8.9. For decelerated rectilinear motion with initial speed of light $c$ in a uniform electric field of magnitude $E$, with mass $m$ dependent on speed and charge $-e$, special reltovity gives speed as: $v=c$
A particle moving at speed $c$, with infinite mass, cannot be stopped by any force. It moves at speed c, gaining potential energy without losing kinetic energy.
9. KINETIC ENERGY GAINED, POTENTIAL ENERGY LOST AND ENERGY RADIATED
9.1. Special relativity gives the kinetic energy $K$ gained by an electron, of mass $m$ and charge $-e$, acelerated in a straight line through a distance $x$ by an electric field $E$, from 0 (zero) initial speed to speed $v$, as: $K=$ $m_{o} c^{2}\left\{\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}}-1\right\}$
Equal to potential energy $P$ lost. 9.2. Potential energy lost is $P=$ $m_{o} c^{2}\left\{\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}}-1\right\}$
$r=\frac{m_{o} v^{2}}{e E}\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}}=\gamma r_{o}$
8.8. For accelerated rectilinear motion with zero initial speed in a uniform electric field of magnitude $E$, with constant mass $m$ and charge $-e$, equation (14C) gives speed $v$ at time $t$ as:
$v=c\left(1-e^{\frac{-a t}{c}}\right)$
where $a=\frac{e E}{m}$ is a constant.
Speed of light $c$ is the maximum attainable, as time $t \rightarrow \infty$
8.9. For decelerated rectilinear motion with initial speed of light $c$ in a uniform electric field of magnitude $E$, with constant mass $m$ and charge $-e$, equation (15C) gives speed $v$ at time $t$ as:
$v=c\left(2 e^{\frac{-a t}{c}}-1\right)$
Speed as $t \rightarrow \infty$, is $-c$ The speed of light $c$ is the maximum attainable.
9. KINETIC ENERGY GAINED, POTENTIAL ENERGY LOST AND ENERGY RADIATED
9.1. Equation (14C) gives the kinetic energy $K$ gained by an electron, of mass $m$ and charge $-e$, acelerated in a straight line through a distance $x$ by an electric field $E$, from 0 (zero) initial speed to speed $v$, as $K=$ $\int_{0}^{x} F(d x)=m \int_{0}^{v} v(d v)=\frac{1}{2} m v^{2}$
(18C)
9.2. Potential energy $P$ lost In a distance $x$ at speed 0 to $v$, is:
$P=e \int_{0}^{x} E(d x)=m \int_{0}^{v} v \frac{d v}{1-\frac{v}{c}}$

| 9.3. Energy radiated ( $P-K$ ) should be: 0 (zero) <br> (20A) | 9.3. Energy radiated ( $P-K$ ) should be: 0 (zero) <br> (20B) | $\left.=-m c^{2} \ln \left(1-\frac{v}{c}\right)-m c v \quad 19 C\right)$ <br> 9.3. Energy radiated: $R_{d}=-m c^{2} \ln \left(1-\frac{v}{c}\right)-m c v-\frac{1}{2} m v^{2}$ |
| :---: | :---: | :---: |
| 10. KINETIC ENERGY LOST, POTENTIAL ENERGY GAINED AND ENERGY RADIATED <br> 10.1. Kinetic energy $K$ lost by an electron of constant mass $m$ decelerated in a straight line from speed of light $c$ to speed $v$ is: $K=\frac{1}{2} m\left(c^{2}-v^{2}\right)$ <br> (21A) <br> 10.2. Equation (15A) gives the potential energy $P$ gained by an electron of constant mass $m$ and charge $-e$, decelerated in a straight line by an electric field $E$, from the speed of light $c$ to speed $v$ as: $\begin{equation*} P=\frac{1}{2} m\left(c^{2}-v^{2}\right) \tag{22A} \end{equation*}$ <br> 10.3. Energy radiated, $K-P$, should be 0 (zero). <br> (23A) | 10. KINETIC ENERGY LOST, POTENTIAL ENERGY GAINED AND ENERGY RADIATED <br> 10.1. An electron moving at the speed of light $c$ with infinite mass cannot be stopped by any retarding force. It moves with speed $c$, gaining potential energy Kinetic energy lost is $K=0 .(21 \mathrm{~B})$ 10.2. The potential energy $P$ gained by an electron of rest mass $m_{0}$ and charge $-e$, decelerated in a straight line by an electric field $E$, through a distance $x$ from speed of light $c$ to speed $v$, is: $\begin{equation*} P=e \int_{0}^{x} E(d x) \tag{22B} \end{equation*}$ <br> 10.3. Energy radiated $K-P$ should be 0 (zero) | 10. KINETIC ENERGY LOST, POTENTIAL ENERGY GAINED AND ENERGY RADIATED <br> 10.1. Kinetic energy $K$ lost by an electron of mass $m$ decelerated from the speed of light $c$ to speed $v$ is: $\begin{equation*} K=\frac{1}{2} m\left(c^{2}-v^{2}\right) \tag{21C} \end{equation*}$ <br> 10.2. Equation ( ${ }^{15 \mathrm{C} \text { ) gives }}$ potential energy $P$ gained by an electron of mass $m$ and charge $e$, decelerated by an electric field $E$, from the speed of Ight $c$ to speed $v$ as equation (22C): $\begin{aligned} & P=e \int_{0}^{x} E(d x)=-m \int_{c}^{v} v \frac{d v}{1+\frac{v}{c}} \\ & P=m c^{2} \ln \frac{1}{2}\left(1+\frac{v}{c}\right)+m c^{2}\left(1-\frac{v}{c}\right) \end{aligned}$ <br> 10.3. Energy radiated: $R_{d}=\frac{1}{2} m\left(c^{2}-v^{2}\right)$ <br> $-m c^{2} \ln \frac{1}{2}\left(1+\frac{v}{c}\right)-m c^{2}\left(1-\frac{v}{c}\right)$ |
| 11. MASS-ENERGY EQUIVALENCE LAW <br> 11.1. Energy content of a particle of mass $m$ is not considered. <br> 11.2. Classical electrodynanics does not reckon with the internal energy content of a particle of mass $m$. <br> 11.3. Kinetic energy $K$ of a body of mass $m$ moving at speed $v$, is: $K=\frac{1}{2} m v^{2}$ | 11. MASS-ENERGY EQUIVALENCE LAW <br> 11.1. Energy content $E_{n}$ of a body of rest mass $m_{0}$, is $m_{0} c^{2}$. 11.2. Total energy $E$ of a particle of mass $m$ moving at speed $v$, is: $E=m c^{2}=m_{o} c^{2}\left(1-\frac{v^{2}}{c^{2}}\right)^{-\frac{1}{2}}$ <br> 11.3. Kinetic energy $K$ of a body of mass $m$ moving at speed $v$, is: $m_{o} c^{2}\left(1-\frac{v^{2}}{c^{2}}\right)^{-\frac{1}{2}}-m_{o} c^{2}$ | 11. MASS-ENERGY EQUIVALENCE LAW <br> 11.1. Energy content $E_{n}$ of a particle of mass $m$ at rest is: $E_{n}=\frac{1}{2} m c^{2}$ <br> 11.2. Total energy $E$ of a particle of mass $m$ moving at speed $v$, is: $E=\frac{1}{2} m\left(c^{2}+v^{2}\right)$ <br> 11.3. Kinetic energy $K$ of a body of mass $m$ moving at speed $v$, is: $K=\frac{1}{2} m v^{2}$ |
| 12. RADIATION REACTION | 12. RADIATION REACTION | 12. RADIATION REACTION |

12.1. Radiation reaction force $\boldsymbol{R}_{f}$ due to an electron of charge -e moving with velocity $\mathbf{v}$ and acceleration $\mathbf{a}$, at time $t$.

$$
\mathbf{R}_{f}=\frac{e^{2}}{6 \pi \varepsilon_{o} c^{3}} \frac{d \mathbf{a}}{d t}
$$

(Abraham-Lorentz formula) $c$ is speed light in a vacuum.
12.2. There should be no radiation force for revolution in a circle of radius $r$ with speed $v$ and constant acceleration $-v^{2} / r$
13. RADIATION POWER
13.1 Radiation power $R_{p}$ of an electron of charge -e moving at speed $v$ and acceleration $\mathbf{a}$, is:
$R_{p}=\frac{e^{2} a^{2}}{6 \pi \varepsilon_{o} c^{3}}$

## (Larmor formula)

where $c$ is the speed of light in a vacuum.
13.2. Radiation power $R_{p}$ of an electron of charge -e revolving with speed $v$ and centripetal acceleration $a=v^{2} / r$, in a circle of radius $r$, is:
$R_{p}=\frac{e^{2} v^{4}}{6 \pi \varepsilon_{o} r^{2} c^{3}}$

## (Larmor formula)

(This formula led to Bohr's quantum mechanics designed to prevent the hydrogen atom from radiating snd collapsing.
14. SPEED OF LIGHT IN A MOVING MEDIUM
Speed of light $w$ at normal incidence in a medium of refractive index $\mu$ moving with speed $v$ in a vacuum.
$w=\frac{c}{\mu}+v\left(1-\frac{1}{\mu}\right)$
15. FRINGE SHIFT IN THE FIZEAU'S EXPERIMENT (1850)
For a path length of $2 L$ in water of refractive index $\mu$ moving with
12.1. the formula for radiation reaction force, in relativistic electrodynamics, has some difficulties.
See David Griffith, Introduction to Electrodynamics, Prentice-Hall Inc., New Jersey, 1981, p. 382
12.2. Without radiation force, there should have been no radiation power.
13. RADIATION POWER
13.1. Radiation power $R_{p}$ of an electron of charge -e moving with speed $v$ and acceleration $\boldsymbol{a}$ in the same direction, is:
$R_{p}=\frac{e^{2} a^{2} \gamma^{4}}{6 \pi \varepsilon_{o} c^{3}}$
(Lienard formula)
where $c$ is speed of light and
Lorentz factor $\gamma=\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}}$
13.2. Radiation power $R_{p}$ of an electron of charge -e revolving with speed $v$ and acceleration a $=v^{2} / r$, in a circle of radius $r$, is
$R_{p}=\frac{e^{2} v^{4} \gamma^{4}}{6 \pi \varepsilon_{o} r^{2} c^{3}}$
(Lienard formula)
(Radiation power $R_{p}$ explodes at speed $v=c$ ).
14. SPEED OF LIGHT IN A MOVING MEDIUM
Speed $w$ at normal incidence in a medium of refractive index $\mu$ moving with speed $v$ in vacuum.
$w=\frac{c}{\mu}+v\left(1-\frac{1}{\mu^{2}}\right)$
(Fresnel's law)
15. FRINGE SHIFT IN THE FIZEAU'S EXPERIMENT (1850)
For a path length of $2 L$ in water of refractive index $\mu$ moving with
12.1. Radiation reaction force $R_{f}$ due to an electron of charge -e moving with velocity v in an electric field of magnitude $E$ and intensity $\mathbf{E}$ is expressed as:
$\mathbf{R}_{f}=\frac{e E}{c}(\mathbf{c}-\mathbf{v})+e \mathbf{E}$
12.2. For linear acceleration in the opposite direction of field $E$ :
$\mathbf{R}_{f}=\frac{e v}{c} \mathbf{E}=-\frac{e E}{c} \mathbf{v}$
where $c$ is the speed of light,
13. RADIATION POWER
13.1 Radiation power of an electron of charge -e moving with velocity $\boldsymbol{v}$, is the scalar product:
$R_{p}=-\mathbf{v} . \mathbf{R}_{f}$
In rectilinear motion $\mathbf{v}$ and $\mathbf{E}$ are
collinear and radiation power is:
$R_{p}=e E \frac{v^{2}}{c}$
Radiation power $R_{p}$ is 0 if $v$ is orthogonal to $E$ and $\mathrm{R}_{f}$ as in circular motion of an electron round a nucleus.
13.2. Radiation power of an electron revolving with speed $v$ and acceleration $a=v^{2} / r$, in a circle of radius $r$, is zero. This makes Rutherford's nuclear model of the hydrogen atom without radiation and stable outside Bohr's quantum mechanics.
14. SPEED OF LIGHT IN A MOVING MEDIUM
Speed of light $w$ at normal incidence in a medium of refractive index $\mu$ moving with speed $v$ in a vacuum.
$w=\frac{c}{\mu}+v\left(1-\frac{1}{\mu}\right)$
15. FRINGE SHIFT IN THE
FIZEAU'S EXPERIMENT (1850)

For a path length of $2 L$ in water of refractive index $\mu$ moving with
speed $v$, fringe shift, with light of wavelength $\lambda$, is :
$\delta=\frac{4 L v \mu^{2}}{\lambda c}\left(1-\frac{1}{\mu}\right)$
16. FRINGE SHIFT IN THE M-M EXPERIMENT (1887)
With relativity of speed between light and a moving mirror, for perpendicular arms of equal length $L$ and speed $v$ along one arm, the fringe shift, with light of wavelength $\lambda$, is: $\delta=\frac{L v^{2}}{\lambda c^{2}}$
17. FRINGE SHIFT IN THE

SAGNAC EXPERIMENT (1913)
With relativity of speed between light and a moving mirror, for a round trip of length 4L and a disc of radius $R$ and angular frequency $\omega$, fringe shift, with light of wavelength $\lambda$, is:

$$
\delta=\frac{8 \omega R L}{\lambda c}=\frac{8 v L}{\lambda c}
$$

18. DOPPLER EFFECT (SHIFT) IN LIGHT PROPAGATION
For an observer following with speed $v$ behind a source moving with speed $u$ and emitting light with speed $c$, a constant relative to the source, Doppler frequency
is: $f_{v}=\frac{c+(v-u)}{c} f_{o}$
Doppler wavelength is:
$\lambda_{v}=\frac{c}{c+(v-u)} \lambda_{o}$
19. DOPPLER EFFECT (SHIFT) IN SOUND PROPAGATION
For an observer following with speed $v$ behind a source moving with speed $u$ and emitting sound with speed $s$, a constant relative to the mediun (air), Doppler frequency and wavelength are:

$$
f_{v}=\frac{s+v}{s+u} f_{o} \text { and } \lambda_{v}=\frac{s+u}{s+v} \lambda_{o}
$$

speed $v$, fringe shift, with light of wavelength $\lambda$, is :

$$
\delta=\frac{4 L v \mu^{2}}{\lambda c}\left(1-\frac{1}{\mu^{2}}\right)
$$

16. FRINGE SHIFT IN THE M-M EXPERIMENT (1887) If relative speed between a light ray and a moving mirror is a constat equal to speed of light $c$, for perpendicular arms of equal length $L$ and speed $v$ along one arm, fringe shift, for light of wavelength $\lambda$,should :be: 0 (zero)

## 17. FRINGE SHIFT IN THE

 SAGNAC EXPERIMENT (1913) If relative speed between a light ray and a moving mirror is $c$, a constat, for a round trip of length 4 L and a disc of radius $R$ rotating with angular frquency $\omega$, fringe shift, with light of wavelength $\lambda$, should be:0 (zero)
18. DOPPLER EFFECT (SHIFT) IN LIGHT PROPAGATION The speed of light being a constant $c$ for all moving observers, accordimg to the theory of special relativity, (constancy of the speed of light) there should be no Doppler shift in frequency or wavelength. That is $f_{v}=f_{0}$
and $\lambda_{v}=\lambda_{0}$,
This is contrary to observations.
19. DOPPLER EFFECT (SHIFT) IN SOUND PROPAGATION
For an observer following with speed $v$ behind a source moving with speed $u$ and emitting sound with speed $s$, a constant relative to the mediun (air), Doppler frequency and wavelength are:
$f_{v}=\frac{s+v}{s+u} f_{o} \lambda_{v}=\frac{s+u}{s+v} \lambda_{o}$
speed $v$, fringe shift, with light of wavelength $\lambda$, is :

$$
\delta=\frac{4 L v \mu^{2}}{\lambda c}\left(1-\frac{1}{\mu}\right)
$$

16. FRINGE SHIFT IN THE M-M EXPERIMENT (1887)
With relativity of speed between light and a moving mirror, for perpendicular arms of equal length $L$ and speed $v$ along one arm, the fringe shift, with light of wavelength $\lambda$, is: $\delta=\frac{L v^{2}}{\lambda c^{2}}$

## 17. FRINGE SHIFT IN THE

 SAGNAC EXPERIMENT (1913) With relativity of speed between light and a moving mirror, for a round trip of length 4 L and a disc of radius $R$ and angular frequency $\omega$, fringe shift, with light of wavelength $\lambda$, is:$$
\delta=\frac{8 \omega R L}{\lambda c}=\frac{8 v L}{\lambda c}
$$

18. DOPPLER EFFECT (SHIFT) IN LIGHT PROPAGATION
For an observer following with speed $v$ behind a source moving with speed $u$ and emitting light with speed $c$, a constant relative to the source, Doppler frequency is: $f_{v}=\frac{c+(v-u)}{c} f_{o}$
Doppler wavelength is:

$$
\lambda_{v}=\frac{c}{c+(v-u)} \lambda_{o}
$$

19. DOPPLER EFFECT (SHIFT) IN SOUND PROPAGATION
For an observer following with speed $v$ behind a source moving with speed $u$ and emitting sound with speed $s$, a constant relative to the mediun (air), Doppler frequency and wavelength are:
$f_{v}=\frac{s+v}{s+u} f_{o}$ and $\lambda_{v}=\frac{s+u}{s+v} \lambda_{o}$

## 4. Graphical Comparison of Equations of Motion

### 4.1. Equations of Speed Versus Time

Figure 2 shows graphs of v/c (speed in units of c) against at/c (time in units of $c / a)$ for an electron of charge $-e$ and mass $m$ and rest mass $m_{0}$ accelerated from zero initial speed or decelerated from the speed of light $c$, by a uniform electric field of magnitude $E$, where $a=e E / m$ is a constant; the lines $A 1$ and $A 2$ (equations 16A and 17A) according to classical electrodynamics, the dashed curve B1 and line B2 (equations 16 B and 17 B ) according to relativistic electrodynamics and the dotted curves C1 and C2 (equations 16C and 17C) according to the alternative electrodynamics.


Figure 2. Graphs of speed versus time for an electron movig in an electric field

### 4.2. Equations of Speed Versus Kinetic Energy

Figure 3 shows graphs of $\mathrm{v} / \mathrm{c}$ (speed in units of c ) against $\mathrm{K} / \mathrm{mc}^{2}$ (kinetic energy, gained or lost, in units of $m c^{2}$ ) for an electron of charge -e and mass $m$ accelerated from zero initial speed or decelerated from the speed of light $c$, by a uniform field of magnitude $E$, where $a=e E / m$ is a constant; the dotted curves ( $A 1 \& C 1$ ) and ( $A 2$ \& C2) (equations 16A and 17A) according to classical electrodynamics and the alternative electrodynamics, the dashed curve B1 (equations 16B and 17B) according to relativistic electrodynamics. No curve is obtainable in relativistic electrodynamics for an electron decelerated from the speed of light.


Figure 3. Graphs of speed versus kinetic energy for an electron movig in an electric field

### 4.3 Equations of Speed Versus Potential Energy

Figure 4 shows graphs of $v / c$ (speed in units of $c$ ) against $P / m c^{2}$ (potential energy, lost or gained, in units of $m c^{2}$ ) for an electron of mass $m$ accelerated from zero initial speed or decelerated from the speed of light $c$, the solid lines A1 and A2 (equations 19A and 22A) according to classical electrodynamics, the dashed curve B1 and line $B 2$ (equations 19B and 22B) according to relativistic electrodynamics and the dotted curves C1 and C2 (equations 19C and 22C) according to the alternative electrodynamics. The solid squares are the result of Bertozzi's experiment (1964).


Figure 4. Graphs of speed versus potential energy gained or lost by an electron movig in an electric field

### 4.4. Equations of Speed Versus Energy Radiation

Figure 5 shows graphs of $v / c$ (speed in units of c) against $R / m c^{2}$ energy radiated (in units of $m c^{2}$ ) for an electron of mass $m$ and charge $-e$ accelerated by an electric field from zero initial speed or decelerated from the speed of light $c$ (equations 20C and 23C) according to the alternative electrodynamics. Energy is always radiated, under acceleration or deceleration.There is no such energy radiation graphs from the point of view of classical electrodynamics or relativistic electrodynamics. Herein lies the difference between the three systems of electrodynamics.


Figure 5. Graphs of speed versus energy radiated by an electron movig in an electric field

## 5. Summary of Forces, Speeds, Energies and Radiation.

Table 2 below gives a summary of forces, speeds, kinetic energies, potential energiess and radiation for an accelerated or decelerated electron of mass $m$ and charge -e moving in a straight line at time $t$ in an electric field ofmagnitude $E$, under classical, relativistic and alternative of electrodynamics

Table 2. Energy Considerations for an accelerated or decelerated Electron

|  | Force, Speed, Energy | A. Classical electrodynamics | B. Relativistic Electrodynamics | C. Alternative Electrodynamics |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Accelerating force | $\begin{gathered} F_{o}=e E=m \frac{d v}{d t} \\ \text { Equation 14A } \end{gathered}$ | $\begin{gathered} F_{o}=e E=\frac{d}{d t}(m v) \\ \text { Equation 14B } \end{gathered}$ | $F=e E\left(1-\frac{v}{c}\right)=m \frac{d v}{d t}$ <br> Equation 14C |
| 2 | Decelerating force | $\begin{gathered} F_{o}=e E=-m \frac{d v}{d t} \\ \text { Equation 15A } \end{gathered}$ | $F_{o}=e E=-\frac{d}{d t}(m v)$ <br> Equation 15B | $F=e E\left(1+\frac{v}{c}\right)=-m \frac{d v}{d t}$ <br> Equation 15 C |
| 3 | Speed $v$ at time $t$ under acceleration from zero initial speed | $v=\frac{e E}{m_{o}} t=a t$ <br> where $m=m_{0}$ <br> Equation 16A | $v=a t\left\{1+\left(\frac{a t}{c}\right)^{2}\right\}^{\frac{-1}{2}}$ <br> Equation 16B | $v=c\left(1-e^{\frac{-a t}{c}}\right)$ <br> where $m=m_{0}$ <br> Equation 16C |
| 4 | Speed $v$ at time $t$ under deceleration from speed $c$. | $v=c-a t$ <br> Equation 17A | c. <br> Equation 17B | $v=c\left(2 e^{\frac{-a t}{c}}-1\right)$ <br> Equation 17C |
| 5 | Kinetic energy $K$ gained by an electron accelerated from 0 to speed $v$. | $\frac{1}{2} m v^{2}$ <br> Equation 18A | $m_{o} c^{2}\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}}-m_{o} c^{2}$ <br> Equation 18B | $\frac{1}{2} m v^{2}$ <br> Equation 18 C |
| 6 | Potential energy P lost by an electron accelerated from 0 to speed $v$. | $\frac{1}{2} m v^{2}$ <br> Equation 19A | $m_{o} c^{2}\left(1-\frac{v^{2}}{c^{2}}\right)^{\frac{-1}{2}}-m_{o} c^{2}$ <br> Equation 19B | $-m c^{2} \ln \left(1-\frac{v}{c}\right)-m c v$ <br> Equation 19C |
| 7 | Energy radiated ( $P$ $K$ ) by an electron accelerated from 0 to speed $v$. | 0 <br> Equation 20A | Equation 20B | $\begin{aligned} & -m c^{2} \ln \left(1-\frac{v}{c}\right)-m c v \\ & -\frac{1}{2} m v^{2} \end{aligned}$ <br> Equation 20C |
| 8 | Kinetic energy $K$ lost by an electron decelerated from speed of light c to speed $v$. | $\frac{1}{2} m\left(c^{2}-v^{2}\right)$ <br> Equation 21A | Equation 21B | $\frac{1}{2} m\left(c^{2}-v^{2}\right)$ <br> Equation 21C |


| 9 | Potential energy $P$ gained by an electron decelerated in distance $x$ from speed of light $c$ to speed $v$. | $\frac{1}{2} m\left(c^{2}-v^{2}\right)$ <br> Equation 22A | $\int_{0}^{x} e E(d x)$ <br> Equation 22B | $\begin{aligned} & m c^{2} \ln \frac{1}{2}\left(1+\frac{v}{c}\right) \\ & +m c^{2}\left(1-\frac{v}{c}\right) \end{aligned}$ $\text { Equation } 22 \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: |
| 10 | Energy radiated $K-P$ by an electron decelerated in distabce $x$ from speed of light c to speed $v$. | $0$ <br> Equation 23A | $0$ <br> Equation 23B | $\begin{aligned} & \frac{1}{2} m\left(c^{2}-v^{2}\right) \\ & -m c^{2} \ln \frac{1}{2}\left(1+\frac{v}{c}\right) \\ & -m c^{2}\left(1-\frac{v}{c}\right) \\ & \text { Equation } 23 \mathrm{C} \end{aligned}$ |
| 11 | Kinetic energy K lost by an electron decelerated from speed of light $c$ to 0 . | $\frac{1}{2} m c^{2}$ | 0 | $\frac{1}{2} m c^{2}$ |
| 12 | Potential energy $P$ gained by an electron decelerated from speed of light $c$ to 0 . | $\frac{1}{2} m c^{2}$ | Infinitely large | $0.307 m c^{2}$ |
| 13 | Energy ( $K-P$ ) radiated by an electron decelerated from speed $c$ to 0 . | 0 | 0 | $0.193 m c^{2}$ |
| 14 | Return speed of electron entering a decelerating field with speed of light $c$. | -c | The electron cannot be stopped by any decelerating field | $-0.594 c$ <br> (Energy radiated in the round trip $=0.324 \mathrm{mc}^{2}$ ) |

## 6. Conclusion

Classical electrodynamics does not reckon with the internal energy content of a particle or body at rest but gives kinetic energy of a particle of constant mass $m$, equal to the rest mass $m_{0}$, moving with speed $v$, as $1 / 2 m v^{2}$. Relativistic electrodynamics, where $m=\gamma m_{0}$, gives the total energy content as $m c^{2}=\gamma m_{0} c^{2}$ and the kinetic energy as $m_{o} c^{2}(\gamma-1)$, where $\gamma$ is the Lorentz factor The alternative electrodynamics gives the total energy content as $1 / 2 m\left(c^{2}+v^{2}\right)$ and kinetic energy as $1 / 2 m v^{2}$. At low speeds, compared to that of light, the three systems of electrodynamics give the same value $\left(1 / 2 m v^{2}\right)$ for kinetic energy. Special relativity gives the energy content of a particle at rest as $m_{0} c^{2}$ whereas the alternative electrodynamics makes it $1 / 2 m_{0} c^{2}$. The difference being a factor of $1 / 2$ notwithstanding, a body of rest mass $m_{0}$ is a source of tremendous amount of energy and radiation..

There is no formula for radiation reaction force in relativistic electrodynamics but there is Lienard formular for radiation power also being proportional to the square of acceleration. In the alternative electrodynamics, radiation reaction force in rectilinear
motion is proportional to velocity and radiation power is the scalar product of radiation reaction force and velocity so that where motion of a charged particle is perpendicular to an electric field, as in circular revolution, the radiation power is zero. This makes Rutherford's nuclear model of the hydrogen atom inherently stable.

Abraham-Lorentz formula giving radiation reaction force due to a charged particle moving in an electric field, as proportional to the rate of change of acceleration, is not consistent with Larmor formula where radiation power is propoprtional to square of acceleration. This is a serious problem in electrodynamics.

The formula for radius of circular revolution $r$ for a charged particle, such as an electron or proton, is the same in relativistic and alternative electrodynamics. If the radius is $r_{0}$ in classical electrodynamics, it becomes $r_{0}$ in relativistic and alternative electrodynamics. It is $\gamma$, the Lorentz factor, which becomes infinitely large at the speed of light. The variation of radius of revolution with $\gamma$ was misconstrued as being the result of mass $m$ increasing with speed, in accordance with special relativity where mass $m=\gamma m_{0}$. Herein lies an explanation for the misconception or delusion of "mass increasing with velocity", claimed to have been accurately supported by laboratory experiments with electrons or protons revolving in a circular orbit.

Most of the experiments supposed to have verified the relativistic mass-velocity formula were performed with electrons or protons revolving in cyclic accelerators. These experiments might as well have confirmed decrease of accelerating force with speed according to the alternative electrodynamics. An experiment is always right but interpretation of the results may be wrong. The relativistic mass-velocity formula, supposed to have been confirmed by experiment, is an expensive case of mistaken identity. This formula is a mathematical contraption, which turned out to be correct for circular revolution, devised to conform to a misinterpretation of experimental results. This misconception must be cleared for modern physics to move forward.

The mass-velocity formula of the theory of special relativity is incidentally correct, to a high degree of accuracy, with respect to circular revolution of a charged particle round a centre of force of attraction, but as a consequence of accelerating force decreasing with velocity and not the result of mass increasing with velocity. Applying this formula to rectilinear motion of a charged particle, under the acceleration of an electric field, is wrong.

In the alternative electrodynamics, an electron of mass $m$ and charge -e moving at the speed of light $c$, on entering a decelerating field of magnitude $E$, is easily brought to rest in time $0.693 \mathrm{mc} / \mathrm{e} E$, after gaining potential energy equal to $0.307 \mathrm{mc}^{2}$ and radiating energy equal to $0.193 m c^{2}$ to come back to the point of entry with speed $-0.594 c$ and it may be accelerated backwards to reach a speed $-c$. In the classical electrodynamics the electron is stopped in time mc/eE after gaining potential energy equal to $0.5 m c^{2}$, to come back to the point of entry with speed $-c$, with no energy radiation. For the relativistic electrodynamics, an electron moving at the speed of light, with an infinitely large mass, cannot be stopped by any field. Electrons in linear accelerators are easily accelerated to the speed of light, with energies over 15 MeV , as demonstrated Bertozzi [9]. A retarding field readily stops such electrons.

Electrons moving at the speed of light being stopped in their tracks and turned back home, constitutes an invalidation of special relativity. It is emission of radiation, the most rampant phenomenon in nature, which makes all the difference between the three systems of electrodynamics. In the alternative electrodynamics there is radiation all the way and this new system may rightly be called radiative electrodynamics.

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